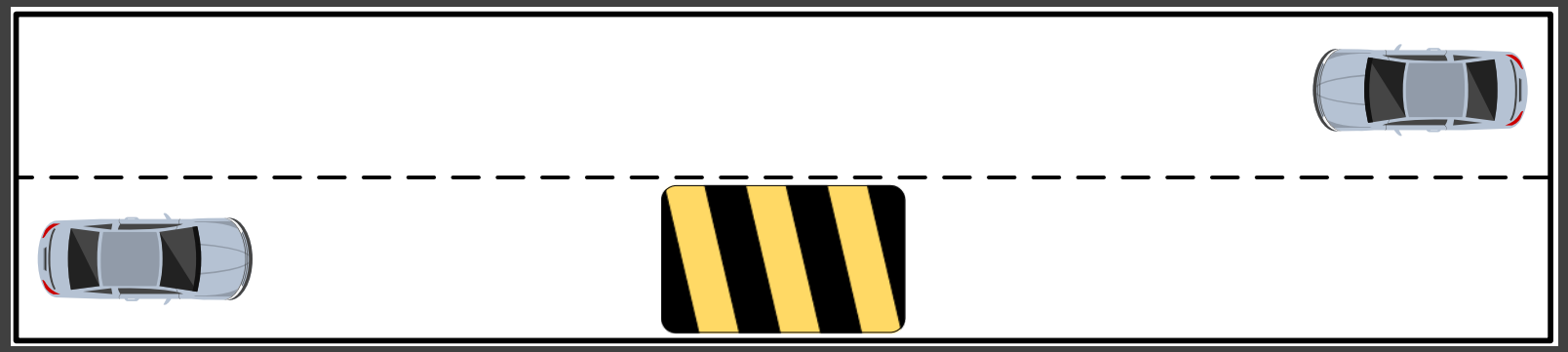
Albin Mosskull and Kaj Munhoz Arfvidsson
Project A1

Supervisor: Elis Stefansson

Avoiding collision with pre-determined route planning

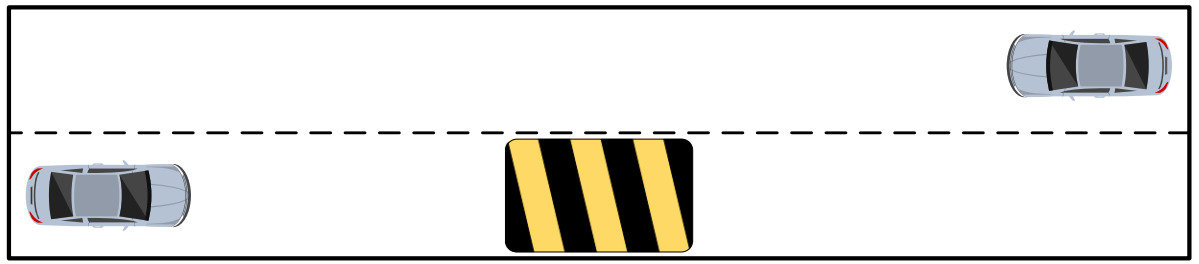


Motivation

Autonomous cars

Route planning

The road sharing problem



Outline

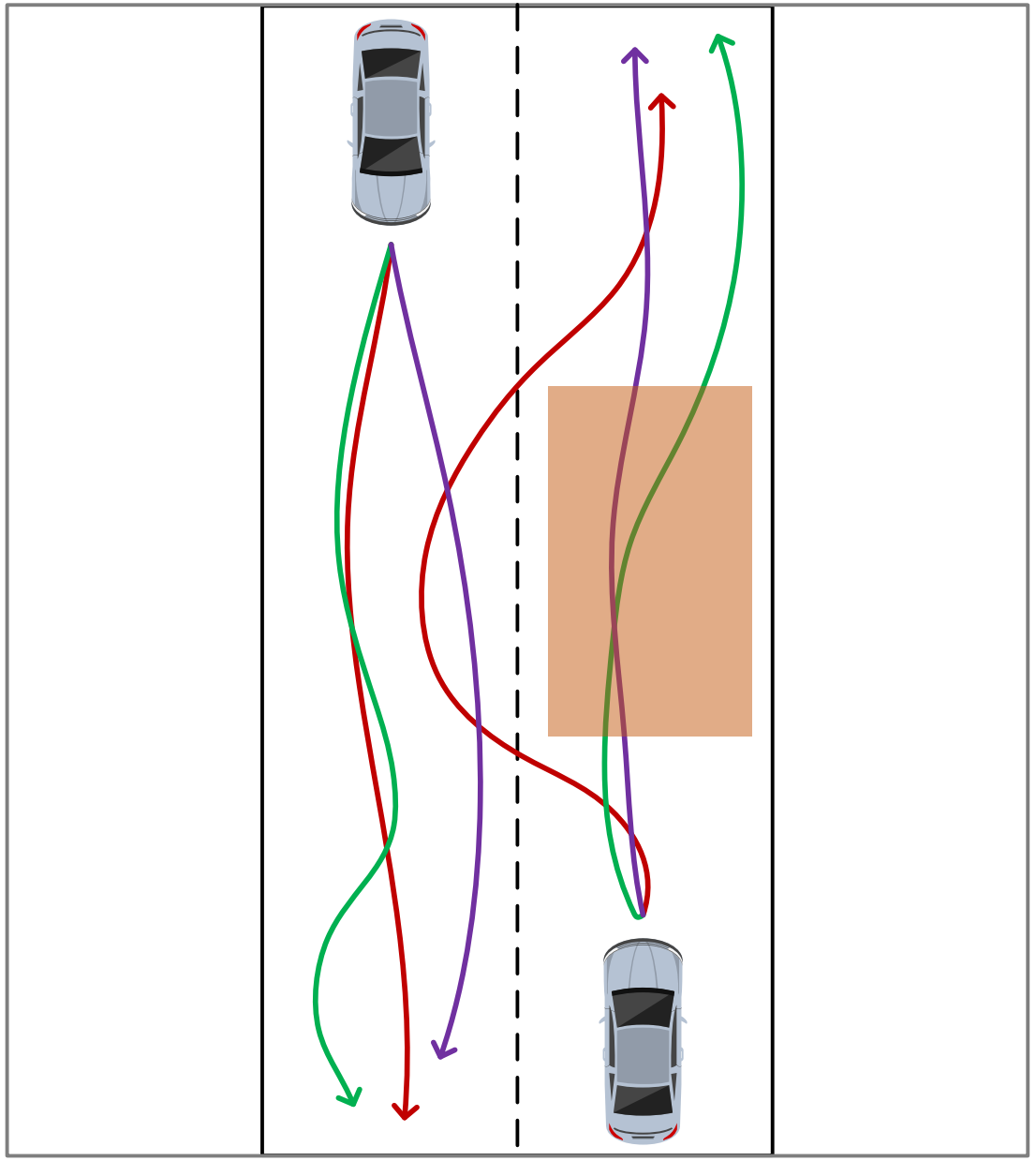
Optimal Control Theory and Tensor Decomposition

Simulations of different traffic situations

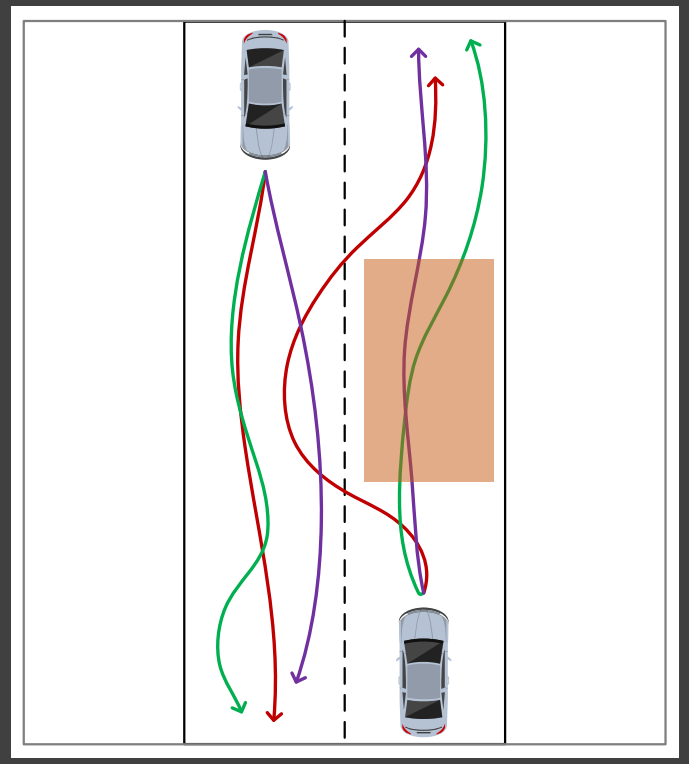
Conclusion with discussion

Theory

Optimal Control Theory



Dynamics



State dynamics:

$$\frac{dx}{dt} = f(t, x, u)$$

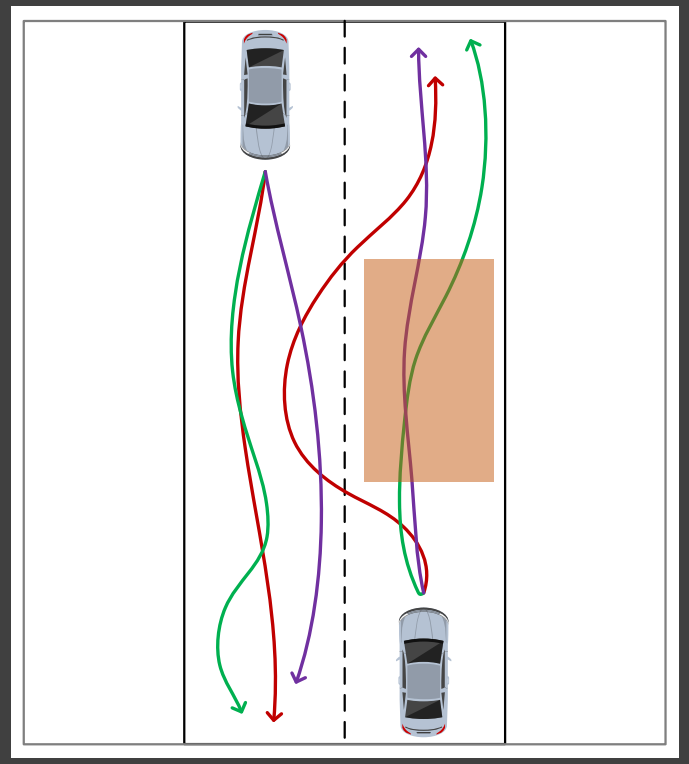
Steering:

moves the car sideway

Gas: moves the car forward

Brake: stops the car

Costs



Cost-to-go function:

$$J(t,x,u) = \phi(x_f) + \int_t^{t_f} c(t,x,u)dt$$

By control:

Fuel

By state:

Traffic rules, Roadblock, Destination

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

$$0 = \min_{u} \left\{ c(x,u) + \frac{\partial}{\partial x} V(x)^T \cdot f(x,u) + \frac{1}{2} Tr\left(\frac{\partial^2}{\partial x^2} V(x) \cdot \sigma(x)\sigma(x)^T \right) \right\}$$

$$\Rightarrow AF = G$$

Problem:

The curse of dimensionality

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

Discretization using tensor decomposition

What is a tensor?

- Multi-dimensional array.
- Generalization of vectors and matrices.

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

Discretization using tensor decomposition

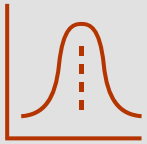
Two alternatives:

- Canonical tensor decomposition
- Tensor Train decomposition

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

Tensor decomposition

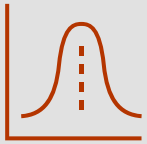


Numerical algorithms

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

Tensor decomposition



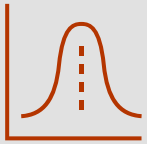
Chosen algorithm:

Sequential Alternating Least Squares

Solving the optimal control problem

Hamilton-Jacobi-Bellman (HJB) Equation

Tensor decomposition

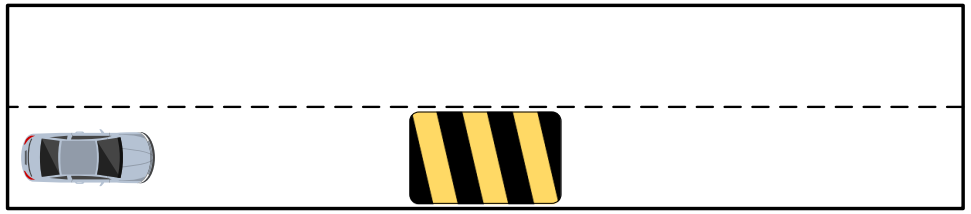


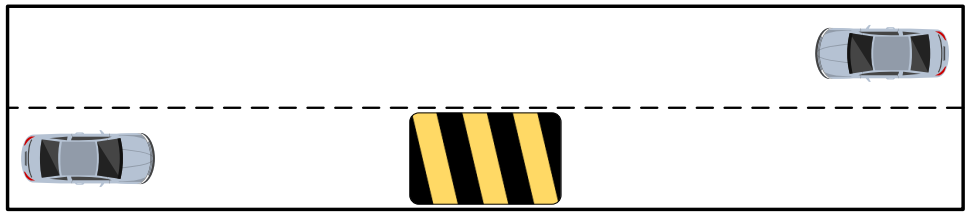
Numerical algorithms

The road sharing problem

Goal:
Reach Destination and Avoid Collision

- A Single Autonomous Vehicle
- Multiple Autonomous Vehicles
- Dynamics of a vehicle

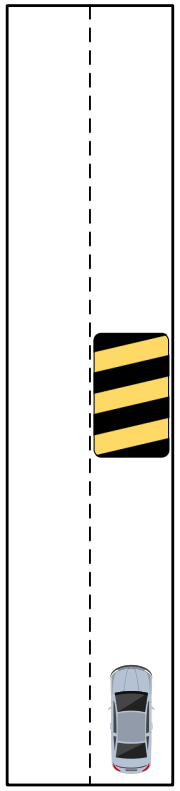


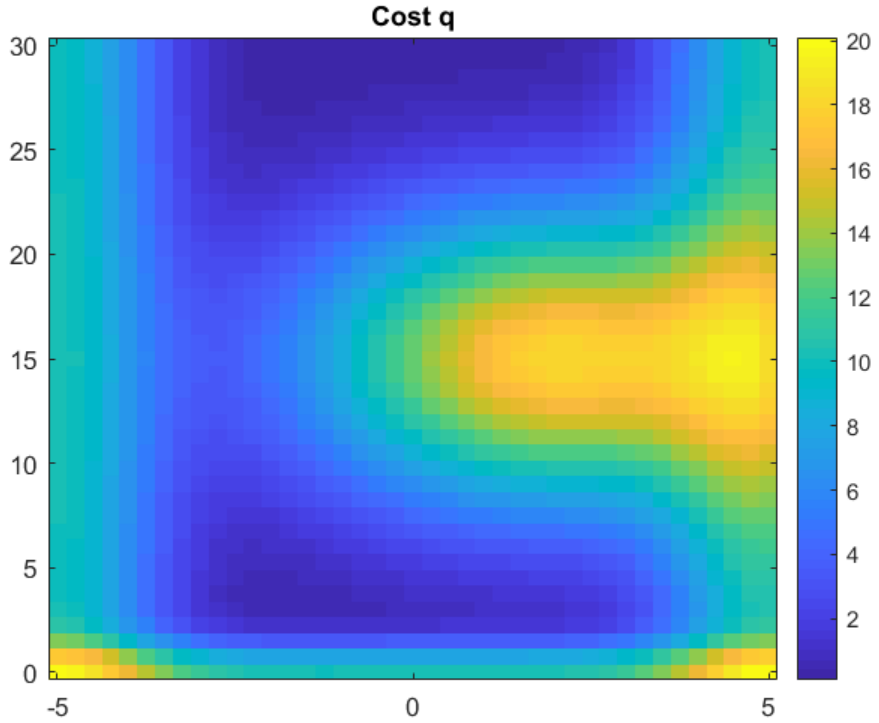


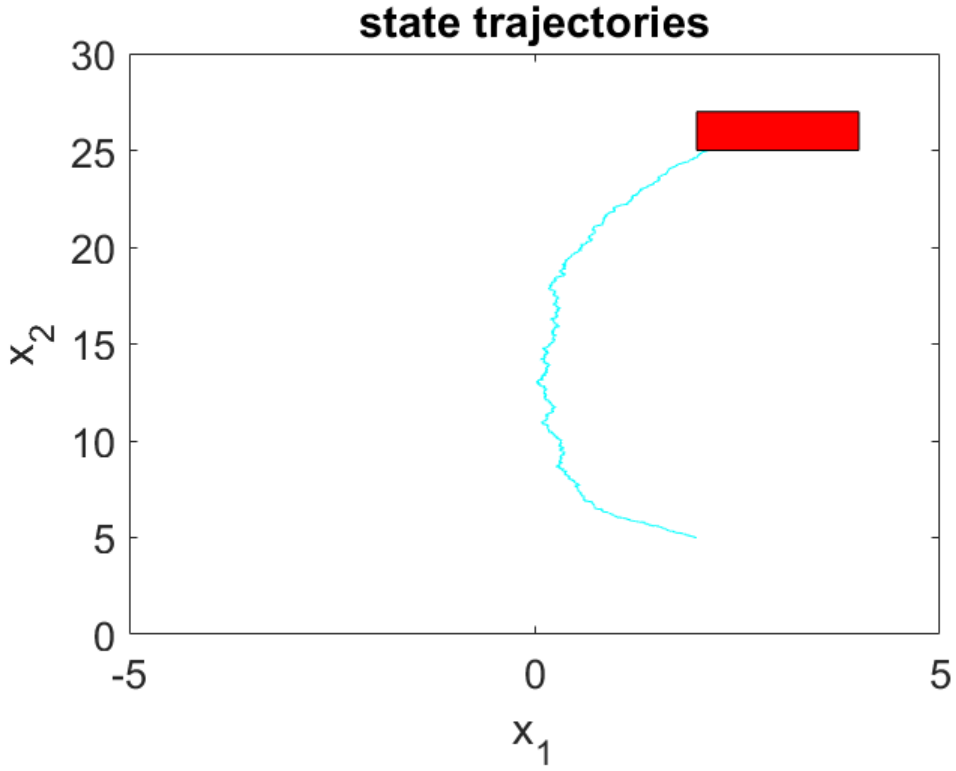
$$dx = \underbrace{\left(0 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right.}_{\text{No impact}} + \underbrace{\left.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right)}_{\text{Control}} dt + \underbrace{\sigma dw}_{\text{Noise}}$$

A Single Autonomous vehicle

- Setting up the costs
- Reaching the destination

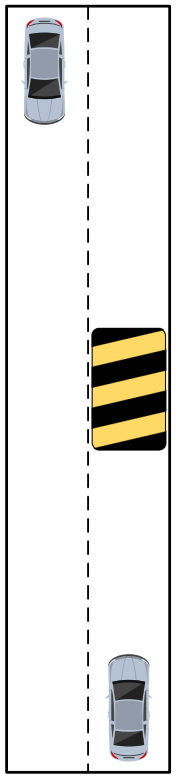


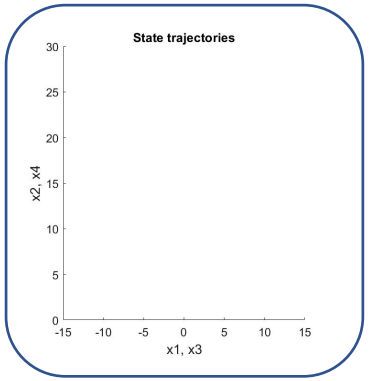


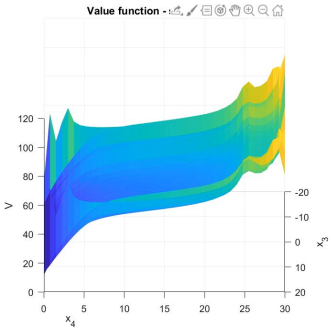


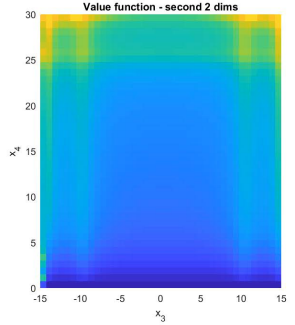
Multiple Autonomous vehicles

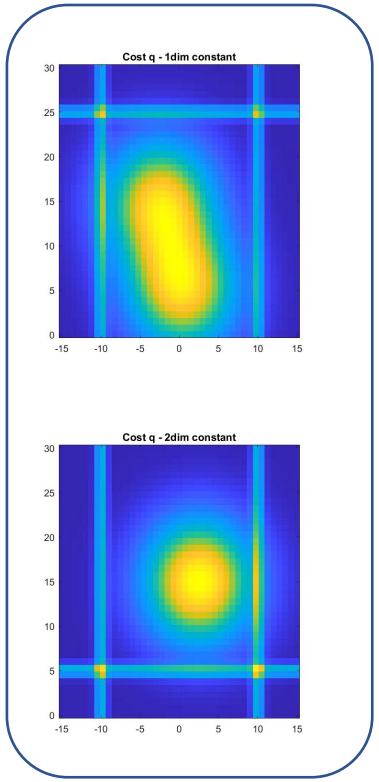
- Setting up the costs
- Car-to-car collision avoidance
- Reaching the destination

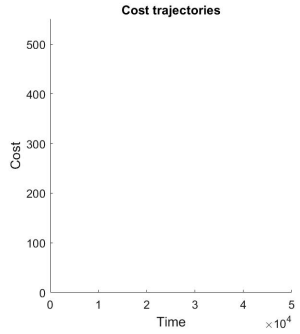


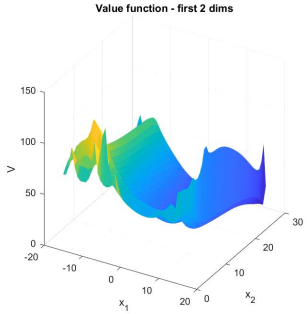


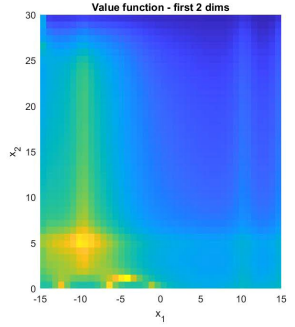






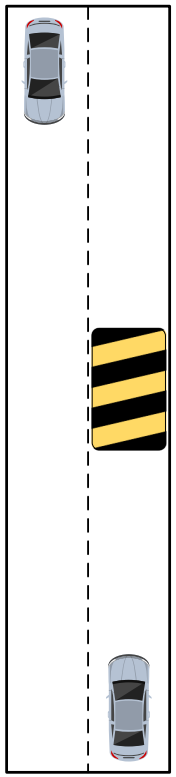


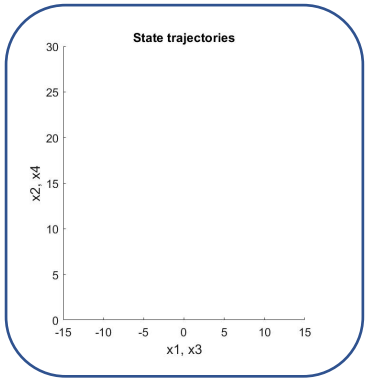


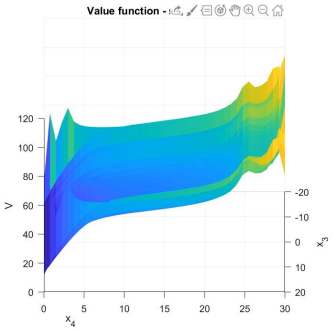


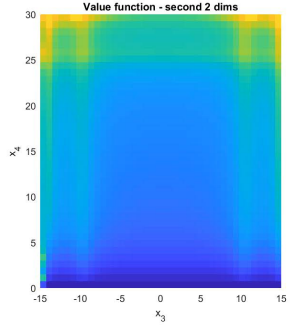
Multiple Autonomous vehicles

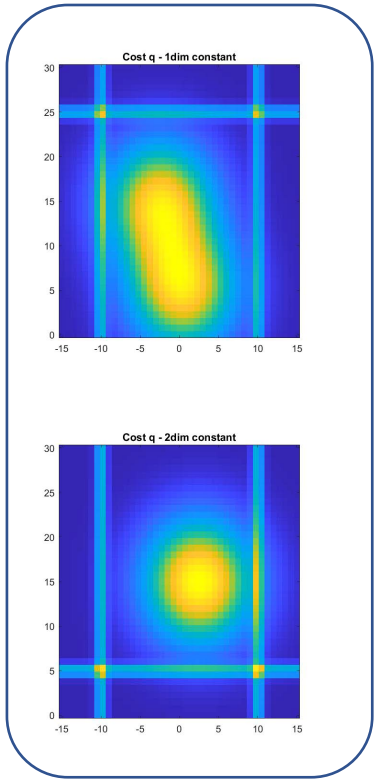
- Setting up the costs
- Car-to-car collision avoidance
- Reaching the destination

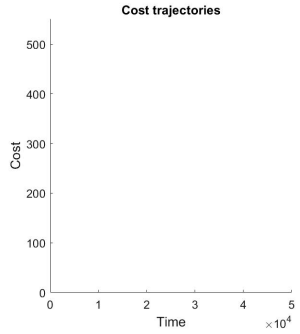


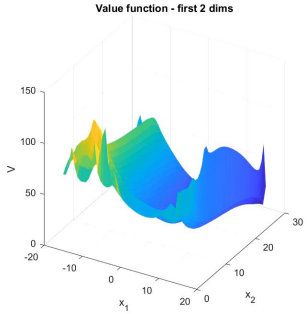


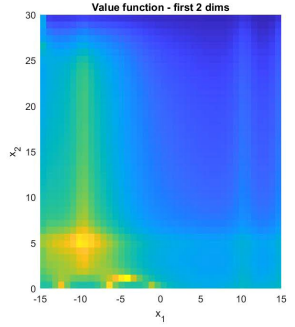












Conclusion

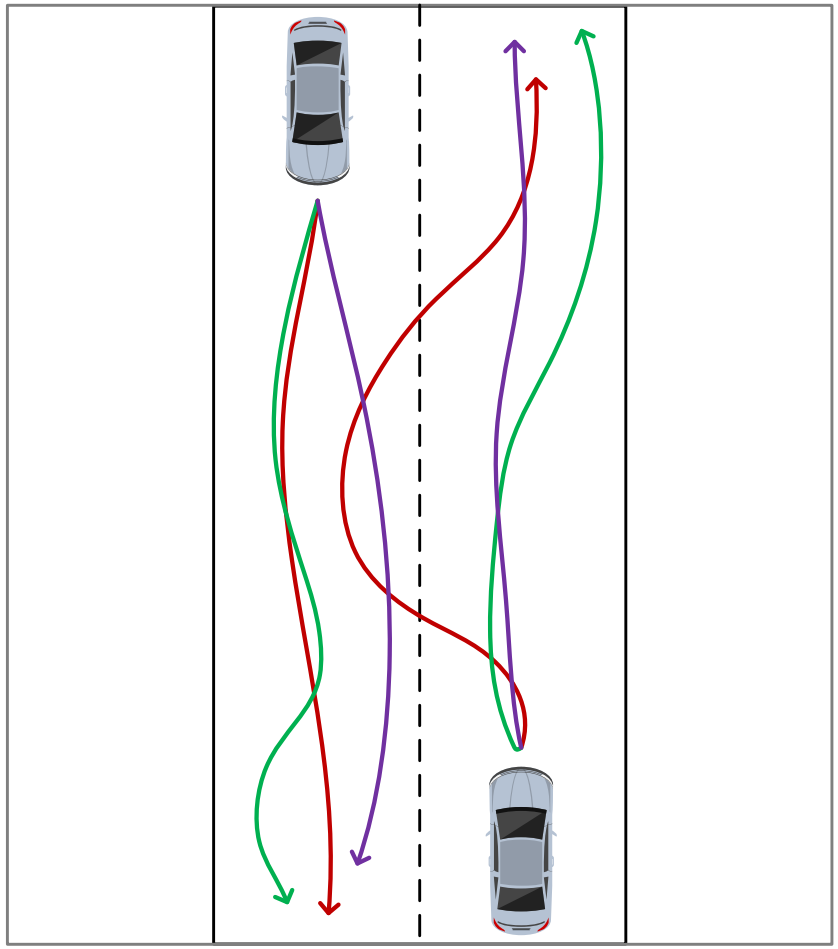
Discussion

Difficult to setup parameters

Numerical errors

Other algorithms could perform better

Once solved, you can reuse the solution



Thank you